STABILITY FOR STOCHASTIC INFINITE-DIMENSIONAL BILINEAR SYSTEMS*

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Abstract. Infinite-dimensional discrete-time bilinear models driven by Hilbert space-valued random sequences can be defined as the uniform limit of finite-dimensional bilinear models. In this paper we shall investigate the limiting properties of the state expectation and correlation sequences for infinite-dimensional stochastic discrete bilinear models, where sufficient conditions for mean square stability will be presented.

Keywords. bilinear systems; discrete systems; infinite-dimensional systems; stability.

1. INTRODUCTION

Stability for stochastic infinite-dimensional continuous-time bilinear systems has been investigated by some authors (e.g. see Zabczyk, 1979; Ichicawa, 1982). In this paper we shall consider the mean square stability problem for infinite-dimensional discrete-time bilinear systems operating in a stochastic environment, whose model is formally given by the following difference equation.

$$x_{i+1} = [A_o + \sum_{k\geq 1} A_k\langle w_i;e_k\rangle]x_i + u_i \ ,$$

where $\{A_k;\ k\geq 0\}$ is a sequence of bounded linear operators on some separable Hilbert space H, $\{e_k;\ k\geq 1\}$ is an orthonormal basis for H, and $\{u_i;\ i\geq 0\}$, $\{w_i;\ i\geq 0\}$ and $\{x_i;\ i\geq 0\}$ are H-valued random sequences. If H is finite-dimensional, then such a model characterizes a finite-dimensional stochastic discrete bilinear system, for which the evolution of the state moments is easy to obtain under independence assumptions (e.g. see Kubrusly, 1985b). Therefore, mean square stability can be investigated by analysing the asymptotic properties of the state moments (e.g. see Kubrusly and Costa, 1985; Kubrusly, 1985b; and the references therein). On the other hand, if $A_k=0$ for every $k\geq 1$, then the above model is naturally reduced to a linear one. In this case the evolution of the state moments are easily obtained, even when H is infinite-dimensional, so that the analysis of their asymptotic behaviour becomes feasible. Actually, some problems related to mean square stability for infinite-dimensional stochastic discrete linear systems have already been properly addressed in the current literature (e.g. see Zabczyk, 1975, 1977; Hager and Horowitz, 1976; Kubrusly, 1985a). The purpose of the present paper is to supply sufficient conditions for mean square stability for the general case, where H is infinite-dimensional and $\{A_k;\ k\geq 0\}$ is any uniformly bounded sequence.

2. NOTATIONAL AND CONCEPTUAL PRELIMINARIES

Throughout this paper we assume that H is a separable nontrivial Hilbert space, with $\|\ \|$ and $\langle;\rangle$ standing for norm and inner product in H, respectively. Let B[X,Y] denote the Banach space of all bounded linear transformations of a Banach space X into a Banach space Y, and set B[X]=B[X,X]. We shall use the same symbol $\|\ \|$ to denote the uniform induced norm in B[X,Y]. Let $C^*\in B[H]$ be the adjoint of $C\in B[H]$, and set $B[H]^+=\{C\in B[H]:\ 0\leq C=C^*\}$, the closed convex cone of all self-adjoint nonnegative (i.e. $0\leq\langle Ch;h\rangle\ \forall h\in H$) operators on H. Let $C^{1/2}\in B[H]^+$ be the (unique) square root of $C\in B[H]^+$, and set $|C|=(C^*C)^{1/2}\in B[H]^+$ for any $C\in B[H]$. The class of all compact operators from B[H] will be denoted by $B_\infty[H]$. If $C\in B_\infty[H]$ (or equivalently, $|C|\in B_\infty[H]$), let $\{\lambda_k\geq 0;\ k\geq 1\}$ be the nonincreasing nonnegative null sequence made up of all singular values of C (i.e. eigenvalues of $|C|$), each nonzero one counted according to its multiplicity as an eigenvalue of $|C|$. Set $\|C\|_1=\sum_{k=1}^{\infty}\lambda_k$ and let $B_1[H]=\{C\in B_\infty[H]:\ \|C\|_1<\infty\}$ be the class of all nuclear (or trace-class) operators on H. Actually $\|\ \|_1$ is a norm in $B_1[H]$ (the so-called trace-norm), and $(B_1[H],\|\ \|_1)$ in a Banach space. We set $B_1[H]^+=B_1[H]\cap B[H]^+$, the class of all correlation operators on H. For any $f,g\in H$ define the outer product operator $(f\circ g)\in B_1[H]$ as follows: $(f\circ g)h=\langle h;g\rangle f$ for all $h\in H$, such that $(f\circ f)\in B_1[H]^+$. The above standard concepts may be found, for instance, in Gohberg and Krein (1969) and Schatten (1970).

Let (Ω,Σ,μ) be a probability space, where Σ is a σ-algebra of subsets of a nonempty basic set Ω, and μ is a probability measure on Σ. Let $\mathcal{H}$ be the set of equivalence classes of H-valued measurable maps x defined almost everywhere (a.e.) on Ω, such that

$$\|x\|^2_{\mathcal{H}} \overset{\text{def.}}{=} \varepsilon\{\|x(\omega)\|^2\} = \int_\Omega \|x(\omega)\|^2 d\mu < \infty ,$$

where ε stands for the expectation operator for scalar-valued random variables. The following inner product in $\mathcal{H}$,

$$\langle x;y\rangle_{\mathcal{H}} \overset{\text{def.}}{=} \varepsilon\{\langle x(\omega);y(\omega)\rangle\} = \int_\Omega \langle x(\omega),y(\omega)\rangle d\mu$$

for all $x,y\in\mathcal{H}$, induces the above norm in $\mathcal{H}$. Thus $\mathcal{H}=L_2(\Omega,\mu;H)$: *the Hilbert space of all second order H-valued random variables*. Given any $x\in\mathcal{H}$ there exists a unique element in H, say E{x}, which is referred to as the *expectation* of $x\in\mathcal{H}$, such that

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$$\langle E\{x\};h\rangle = \varepsilon\{\langle x(\omega);h\rangle\} \quad \forall h\in H \quad .$$

Moreover, for any $x,y\in H$ there exists a unique operator in $B_1[H]$, say $E\{x\circ y\}$, referred to as the *correlation* of $x\in H$ and $y\in H$, such that

$$\langle E\{x\circ y\}f;g\rangle = \varepsilon\{\langle f;y(\omega)\rangle \langle x(\omega);g\rangle\} \ \forall f,g\in H \quad .$$

Note that $\| E\{x\circ y\}\|_1 \le \| x\|_H \| y\|_H$, $E\{x\circ x\}\in B_1[H]^+$, and $\| E\{x\}\|^2 \le \| E\{x\circ x\}\|_1 = \| x\|_H^2$ for every $x,y\in H$. Now consider a family $\{x_\xi\in H;\ \xi\in\Xi\neq\emptyset\}$ of random variables. For each $\xi\in\Xi$ let $\{e_{\xi,k};\ k\ge 1\}$ be an orthonormal basis for H made up of all eigenvectors of $E\{x_\xi\circ x_\xi\}\in B_1[H]^+$, whose existence is ensured by the Spectral Theorem. Such a family is said to be *structurally similar* if there exists an orthonormal basis for H, say $\{e_k;\ k\ge 1\}$, such that $\{e_{\xi,k};\ k\ge 1\}=\{e_k;\ k\ge 1\}$ for every $\xi\in\Xi$. $\{e_k;\ k\ge 1\}$ is referred to as the *common orthonormal basis* for H of $\{x_\xi\in H;\ \xi\in\Xi\}$. Note that structural similarity may be thought of as generalization of correlation stationarity. Actually, a family $\{x_\xi\in H;\ \xi\in\Xi\neq\emptyset\}$ is *correlation stationary* iff there exists $Q\in B_1[H]^+$ such that $E\{x_\xi\circ x_\xi\}=Q$ for every $\xi\in\Xi$, and it is *expectation stationary* iff there exists $q\in H$ such that $E\{x_\xi\}=q$ for every $\xi\in\Xi$. For any family $\{x_\xi\in H;\ \xi\in\Xi\neq\emptyset\}$ we set

$$I_{\{x_\xi;\xi\in\Xi\}} = \{y\in H\text{: y is independent of } \{x_\xi\in H;\xi\in\Xi\}\}.$$

In particular, for any $x\in H$,

$$I_x = \{y\in H\text{: y is independent } x\in H\}.$$

Finally, we shall also need in the sequel the following auxiliary result, which involves the concept of Cauchy summable sequence: a Cauchy sequence $\{y_i\in Y;\ i\ge 0\}$ in a normed linear space Y is *Cauchy summable* iff $\sum_{i=0}^{\infty}\sup_{\nu\ge 0}\| y_{i+\nu}-y_i\| < \infty$.

Proposition 1: Let $\Lambda\in B[X]$ be uniformly asymptotically stable and $\Theta: X\to X$ be a proper contraction on a Banach space X, such that

$$\| \Lambda^i\| \le \sigma\alpha^i \quad \forall i\ge 0 \ ,$$

$$\| \Theta x - \Theta y\| \le \rho\| x - y\| \quad \forall x,y\in X \ ,$$

$$\alpha + \sigma\rho < 1 \ ,$$

for some real constants $\sigma\ge 1$, $0<\alpha<1$, $0\le\rho<1$. If $\{v_i\in X;\ i\ge 0\}$ is a Cauchy summable sequence then $\{z_i\in X;\ i\ge 0\}$, given by

$$z_{i+1} = \Lambda z_i + \Theta z_i + v_i \qquad z_o\in X \ \text{ arbitrary} \ ,$$

is also Cauchy summable, and its limit $z\in X$ does not depend on $z_o\in X$.

Proof: See Kubrusly (1985b).

3. AUXILIARY RESULTS

The purpose of this section is to define properly the infinite-dimensional stochastic discrete bilinear model that has been formally introduced in section 1, and to present the expectation and correlation evolution for the state sequence generated by such a model.

Lemma 1: Let $\{w_i\in H;\ i\ge 0\}$ be a structurally similar sequence with a common orthonormal basis $\{e_k;\ k\ge 1\}$ for H, and let $\{A_k\in B[H];\ k\ge 0\}$ be an uniformly bounded sequence of operators. Set

$$A_{w_i} = A_o + \sum_{k=1}^{\infty} A_k\langle w_i;e_k\rangle : I_{w_i} \to H$$

for every $i\ge 0$, where the above series converges uniformly. That is, for each $i\ge 0$,

$$\sup_{0\neq v\in I_{w_i}} \frac{\| A_{w_i}v - [A_o + \sum_{k=1}^{n} A_k\langle w_i;e_k\rangle]v\|_H}{\| v\|_H} \to 0$$

as $n\to\infty$.

Given $x_o\in H$ and $\{u_i\in H;\ i\ge 0\}$, assume further that $x_o\in I_{x_o}$ and

$$w_j \in I_{\{x_o,u_o,\ldots,u_{j-1},w_o,\ldots,w_{j-1}\}}$$

for every $j\ge 1$. Then the difference equation in H

$$x_{i+1} = [A_o + \sum_{k=1}^{\infty} A_k\langle w_i;e_k\rangle]x_i + u_i$$

has a unique solution, which lies in I_{w_i} for every $i\ge 0$, given by $x_1=A_{w_o}x_o+u_o$ and, for every $i\ge 2$,

$$x_i = A_{w_{i-1}}\cdots A_{w_o}x_o + \sum_{j=1}^{i-1} A_{w_{i-1}}\cdots A_{w_j}u_{j-1} + u_{i-1}.$$

Proof: See Kubrusly (1986).

Lemma 2: Let $\{w_i\in H;\ i\ge 0\}$ be a structurally similar sequence with a common orthonormal basis $\{e_k;\ k\ge 1\}$ for H, and let $\{A_k\in B[H];\ k\ge 0\}$ be uniformly bounded. Given $x_o\in H$ and $\{u_i\in H;\ i\ge 0\}$, assume further that $w_o\in I_{x_o}$ and

$$w_j \in I_{\{x_o,u_o,\ldots,u_{j-1},w_o,\ldots,w_{j-1}\}}$$

for every $j\ge 1$. Now consider the state sequence $\{x_i\in I_{w_i};\ i\ge 0\}$ generated by the difference equation

$$x_{i+1} = [A_o + \sum_{k=1}^{\infty} A_k\langle w_i;e_k\rangle]x_i + u_i \ ,$$

with

$$A_{w_i} = A_o + \sum_{k=1}^{\infty} A_k\langle w_i;e_k\rangle : I_{w_i} \to H$$

defined as in Lemma 1 for each $i\ge 0$. We claim that the state expectation $\{E\{x_i\}\in H;\ i\ge 0\}$ and correlation $\{E\{x_i\circ x_i\}\in B_1[H]^+;\ i\ge 0\}$ sequences evolve according to the following difference equations:

(a) $E\{x_{i+1}\} = F_{w_i}E\{x_i\} + E\{u_i\}$

(b) $E\{x_{i+1}\circ x_{i+1}\} = F_{w_i}E\{x_i\circ x_i\}F^*_{w_i} + T_{w_i}[E\{x_i\circ x_i\}]$

$$+ E\{A_{w_i}x_i\circ u_i\} + E\{A_{w_i}x_i\circ u_i\}^* + E\{u_i\circ u_i\}$$

with $\{F_{w_i}\in B[H];\ i\ge 0\}$ and $\{T_{w_i}\in B[B[H]];\ i\ge 0\}$ given by

$$F_{w_i} = A_o + \sum_{k=1}^{\infty} A_k\langle E\{w_i\};e_k\rangle \ ,$$

$$T_{w_i}[Q] = \sum_{k,\ell=1}^{\infty} \langle (E\{w_i\circ w_i\} - E\{w_i\}\circ E\{w_i\})e_\ell;e_k\rangle A_k Q A^*_\ell$$

for all $Q\in B[H]$, where the above convergence is in terms of the uniform operator norm topdogy (i.e. the above series converge in $B[H]$ and in $B[B[H]]$, respectively). Moreover, if $x_o\in I_{\{u_o,w_o\}}$ and

$$\{u_j,w_j\} \text{ is independent of } \{x_o,u_o,\ldots,u_{j-1},w_o,\ldots,w_{j-1}\}$$

for every $j\ge 1$, which implies the above independence assumption, then

(c) $E\{x_{i+1} \circ x_{i+1}\} = F_{w_i} E\{x_i \circ x_i\} F^*_{w_i} +$

$$+ T_{w_i}[E\{x_i \circ x_i\}] + V^{u_i}_{w_i}[E\{x_i\}]$$

where $\{V^{u_i}_{w_i}: H \rightarrow B_1[H];\ i \geq 0\}$ is defined by the formula

$$V^{u_i}_{w_i}[h] = P^{u_i}_{w_i}[h] + P^{u_i}_{w_i}[h]^* + E\{u_i \circ u_i\}$$

for all $h \in H$, with $\{P^{u_i}_{w_i} \in B[H, B_1[H]];\ i \geq 0\}$ given by

$$P^{u_i}_{w_i}[h] = A_o h \circ E\{u_i\} + \sum_{k=1}^{\infty} A_k h \circ E\{u_i \circ w_i\} e_k$$

for all $h \in H$, where the above series converges uniformly (i.e. it converges in $B[H, B_1[H]]$).

Proof: See Kubrusly (1986).

4. MAIN RESULTS

Consider an infinite-dimensional discrete bilinear model evolving in a stochastic environment as defined in Lemma 1. In this section we shall be interested in the asymptotic behaviour of the state expectation and correlation sequences, whose evolution was given in Lemma 2. In particular, we shall investigate sufficient conditions on the maps $\{A_{w_i}\}$ to ensure that those sequences converge for any admissible initial condition x_o and input disturbance $\{u_i\}$, and their limits do not depend on x_o.

Assumption 1: In order to reach a proper balance between generality of results and simplicity of analysis we make the following assumptions. Let $\{w_i \in H;\ i \geq 0\}$ be an expectation and correlation stationary sequence, and set

$$s = E\{w_i\} \in H \quad , \quad S = E\{w_i \circ w_i\} \in B_1[H]^+ \ ,$$

for every $i \geq 0$. Let $\{e_k;\ k \geq 1\}$ be the orthonormal basis for H made up of all eigenvectors of $S \in B_1[H]^+$, and let $\{A_k \in B[H];\ k \geq 0\}$ be an arbitrary uniformly bounded sequence. Given $x_o \in H$ and $\{u_i \in H;\ i \geq 0\}$, assume that $x_o \in I_{\{u_o, w_o\}}$ and

$\{u_j, w_j\}$ is independent of $\{x_o, u_o, \ldots, u_{j-1}, w_o, \ldots, w_{j-1}\}$

for every $j \geq 1$. Set, for every $i \geq 0$,

$$r_i = E\{u_i\} \in H \quad ,$$

$$R_i = E\{u_i \circ u_i\} \in B_1[H]^+ \quad ,$$

$$G_i = E\{u_i \circ w_i\} \in B_1[H] \quad ,$$

and suppose $G_i = \gamma_i G_o$ for some scalar sequence $\{\gamma_i \in \mathbb{C};\ i \geq 0\}$. Assume further that $\{\gamma_i \in \mathbb{C};\ i \geq 0\}$, $\{r_i \in H;\ i \geq 0\}$, and $\{R_i \in B_1[H]^+;\ i \geq 0\}$ are Cauchy summable sequences in $\mathbb{C}$, H, and $B[H]$, respectively; and $\sup_{i \geq 0} \| R_i \|_1 < \infty$.

Now, under Assumption 1 and according to Lemma 1, consider the state sequence $\{x_i \in I_{w_i};\ i \geq 0\}$ generated by the difference equation

$$x_{i+1} = [A_o + \sum_{k=1}^{\infty} A_k \langle w_i; e_k \rangle] x_i + u_i \quad , \qquad (1)$$

and set for every $i \geq 0$

$$q_i = E\{x_i\} \in H \ , \quad Q_i = E\{x_i \circ x_i\} \in B_1[H]^+ \ .$$

By Lemma 2 it follows that, for every $i \geq 0$,

$$q_{i+1} = Fq_i + r_i \qquad (2)$$

$$Q_{i+1} = FQ_i F^* + T[Q_i] + V_i[q_i] \qquad (3)$$

with $F, M \in B[H]$, $T \in B[B[H]]$, $\{P_i \in B[H, B_1[H]];\ i \geq 0\}$ and $\{V_i: H \rightarrow B_1[H];\ i \geq 0\}$ given by

$$F = A_o + M \ , \quad M = \sum_{k=1}^{\infty} A_k \langle s; e_k \rangle \ ,$$

$$T[Q] = \sum_{k,\ell=1}^{\infty} \langle (S - s \circ s) e_\ell; e_k \rangle A_k Q A^*_\ell \qquad \forall Q \in B[H] \ ,$$

$$P_i[h] = A_o h \circ r_i + \sum_{k=1}^{\infty} A_k h \circ G_i e_k \ , \qquad \forall h \in H \ ,$$

$$V_i[h] = P_i[h] + P_i[h]^* + R_i \ , \qquad \forall h \in H \ ,$$

according to Lemma 2. Note that, if $A_o = 0$ or $M = 0$, then

$$FQF^* + T[Q] = \sum_{k=0}^{\infty} \lambda_k A_k Q A^*_k \qquad \forall Q \in B[H] \qquad (4)$$

where $\lambda_o = 1$ and $\lambda_k \geq 0$ is the eigenvalue of $S \in B_1[H]^+$ associated with the eigenvector e_k for each $k \geq 1$.

Definition 1: The infinite-dimensional stochastic discrete bilinear model in (1) is *mean square stable* if, for any initial condition $x_o \in H$ and input disturbance $\{u_i \in H;\ i \geq 0\}$ satisfying Assumption 1, there exists $q \in H$ and $Q \in B_1[H]^+$ independent of $x_o \in H$, such that

(a) $\| q_i - q \| \rightarrow 0$ as $i \rightarrow \infty$,

(b) $\| Q_i - Q \| \rightarrow 0$ as $i \rightarrow \infty$.

Theorem 1: Consider the evolution equations (2) and (3). If there exist real constants $\sigma \geq 1$ and $0 < \alpha < 1$ such that

$$\| F^i \| \leq \sigma \alpha^i \quad \forall i \geq 0 \quad \text{and} \quad \alpha^2 + \sigma^2 \| T \| < 1 \ ,$$

then the model in (1) is mean square stable.

Sketch Proof: First consider equation (2). Since $\{r_i \in H;\ i \geq 0\}$ is Cauchy summable, it follows from Proposition 1 that Definition 1(a) is satisfied, with $\{q_i \in H;\ i \geq 0\}$ being a Cauchy summable sequence. Now consider equation (3). Set $\Theta = T \in B[B[H]]$ and $\Lambda[Q] = FQF^*$ for all $Q \in B[H]$, where $\Lambda \in B[B[H]]$ is such that

$$\| \Lambda^i \| = \| F^i \|^2 \leq \sigma^2 \alpha^{2i} \qquad \forall i \geq 0 \ .$$

Thus we have from Proposition 1 that $\{Q_i \in B_1[H]^+;\ i \geq 0\}$ is a Cauchy summable sequence in $B[H]$, whose limit $Q \in B[H]$ does not depend on the initial condition, whenever $\{V_i[q_i] \in B_1[H];\ i \geq 0\}$ is Cauchy summable in $B[H]$. Moreover, $Q \in B[H]^+$ since $B[H]^+$ is closed in $B[H]$. Hence Definition 1(b) is satisfied since (cf. Kubrusly, 1986): from the Cauchy summability of $\{\gamma_i \in \mathbb{C};\ i \geq 0\}$, $\{q_i \in H;\ i \geq 0\}$, $\{r_i \in H;\ i \geq 0\}$, and $\{R_i \in B[H];\ i \geq 0\}$ it follows the Cauchy summability of $\{V_i[q_i] \in B[H];\ i \geq 0\}$; and from the uniform boundedness of $\{\| R_i \|_1 ;\ i \geq 0\}$ it can be shown that the uniform limit $Q \in B[H]^+$ is actually nuclear (i.e. $Q \in B_1[H]^+$).

Theorem 2: Consider the evolution equations (2) and (3). Suppose $M = 0$, such that equation (4) also holds true. For each $\ell \geq 0$ let $\Lambda_\ell, \Theta_\ell \in B[B[H]]$ be given by

$$\Lambda_\ell[Q] = \lambda_\ell A_\ell Q A^*_\ell \ , \qquad \Theta_\ell[Q] = \sum_{\substack{k=0 \\ k \neq \ell}}^{\infty} \lambda_k A_k Q A^*_k$$

for all Q B[H], and let $\sigma_\ell \geq 1$ and $\alpha_\ell \geq 0$ be real constants such that

$$\| \Lambda_\ell^i \|^{1/2} = \lambda_\ell^{i/2} \| A_\ell^i \| \leq \sigma_\ell \alpha_\ell^i \quad \forall i \geq 0 \quad .$$

If $\inf_{\ell \geq 0}(\alpha_\ell^2 + \sigma_\ell^2 \| \Theta_\ell \|) < 1$, then the model in (1) is mean square stable.

Sketch Proof: It is readily verified that

$$\lambda_\ell \| A_\ell \|^2 \leq \| \Theta_m \|$$

whenever $\ell \neq m$. If $\inf_{\ell \geq 0}(\alpha_\ell^2 + \sigma_\ell^2 \| \Theta_\ell \|) < 1$, then there exists $m \geq 0$ such that $\alpha_m < 1$ and $\| \Theta_m \| < 1$, since $\sigma_m \geq 1$. Moreover, M=0 implies that

$$\| F^i \| \leq \sigma_o \alpha_o^i \qquad \forall i \geq 0 \quad ,$$

$$\| F \|^2 \leq \| \Theta_m \| \quad \text{if } m > 0 \qquad .$$

Therefore $\| F^i \| \leq \sigma \alpha^i$ for every $i \geq 0$, with $\sigma = \sigma_o \geq 1$ and $0 < \alpha < 1$, where either $\alpha = \alpha_o$ if m=0 or $\| \Theta \|^{1/2} \leq \alpha < 1$ if m>0. Now consider equation (2), and recall that equation (3) can be written as

$$Q_{i+1} = \Lambda_m[Q_i] + \Theta_m[Q_i] + V_i[q_i] \quad ,$$

according to equation (4). Then the desired result follows from Proposition 1 exactly as in the proof of Theorem 1.

5. CONCLUSIONS

In this paper we have established sufficient conditions for mean square stability of infinite-dimensional discrete-time bilinear systems driven by H-valued second order random sequences. The stochastic environment under consideration was characterized by independence and structural similarity with arbitrary and unknown probability distributions.

The main results appeared in Theorems 1 and 2, which were supported by Lemmas 1 and 2. Existence and uniqueness of solutions for infinite-dimensional stochastic discrete bilinear models were considered in Lemma 1, and the evolution of the state expectation and correlation sequences was presented in Lemma 2. The mean square stability conditions supplied in Theorems 1 and 2 extended to infinite-dimensional models the finite-dimensional results proposed by Kubrusly (1985b).

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