A SURVEY ON OPTIMAL SENSORS AND CONTROLLERS LOCATION IN DPS

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Abstract. This paper presents a survey for the optimal sensors and/or controllers location problem in dynamical distributed parameter systems modelled by partial differential equations. The more recent contributions in this field are grouped according to the main goal for which the location problem is developed, namely: system identification, state estimation, and optimal control. In order to pose the sensors and controllers location problem, the semigroup approach for modelling distributed linear systems is briefly reviewed together with its equivalent (infinite-dimensional) and approximate (finite-dimensional) Fourier expansion representations. After presenting a concise general review of the several methods considered in the current literature, it is also proposed a classification of methods. The main classifying factor concerns the use of N-modal approximation schemes, and the different stages of the optimization procedure in which they are required.

Keywords. Controllers location; distributed parameter systems; identification; optimal control; optimization; sensors location; state estimation.

1. INTRODUCTION

The sensors and controllers location is an important problem towards identification, state estimation, and control in distributed systems. In this paper the several methods proposed for solving such a problem are reviewed and classified according to their main characteristics. To begin with it is advisable to pose some abbreviations which will be of frequent usage throughout the text:

ODE: Ordinary Differential Equation(s).
PDE: Partial Differential Equation(s).
LPS: Lumped Parameter System(s).
DPS: Distributed Parameter System(s).
OCL: Optimal Controllers Location.
OSL: Optimal Sensors Location.

DPS, as discussed in this paper, will stand for a dynamical system governed by PDE (opposed to LPS which is described by ODE), as a class of dynamical systems modelled in infinite-dimensional state space (eg. see Helton [39], Ray [65], Curtain and Pritchard [32], and Pritchard [62]). Therefore, it seems reasonable to say that the starting point for analysing DPS is the PDE literature. Presently the available literature in PDE is certainly huge. To mention just a few books, ranging from introductory to advanced texts, published over the past two decades, see for example Courant and Hilbert [26], Garabedian [36], Friedman [35], Mikhlin [55], Treves [73], Showalter [70], and Gustafson [38].

Three of the main problems in system theory (and in particular in DPS)are system identification, state estimation,and optimal control (cf. Ray and Lainiotis, Ed. [66]). In this paper, OCL and OSL will be regarded as intermediate problems for considering the above-mentioned "final" problems. Although little literature has been written in DPS identification compared with what has been done in state estimation and optimal control, some survey papers have already appeared in this field. For instance, see Goodson and Polis [37,61], Kubrusly [46], Ruberti, Ed. [68], Burger and Chavent [19], and Chavent [24]. On the other hand the current literature on state estimation is much richer. For some complete books and surveys on the state estimation problem in DPS, regarding both theory and applications, see for example Bensoussan [14], Phillipson [60], Curtain and Pritchard [32], Sawaragi, Soeda and Omatu [69], Curtain [28], Ray [64], Tzafestas [75], and Bencala and Seinfeld [13]. The optimal control problem in DPS has also been reported in several books and surveys. For instance, see Wang [76], Lions [51-53], Butkoviski [20], Balakrishnam [12], Curtain and Pritchard [32], Robinson [67], Curtain [29], and Bensoussan [16].

The present paper is organized as follows. In section 2 the semigroup approach for modelling DPS is briefly reviewed. After describing a linear model for DPS in a separable Hilbert space, and its equivalent (infinite-dimensional) representation in terms of Fourier expansion, it is also presented the so-called N-modal (finite-dimensional) approximation. The OCL and OSL problems are motivated by the model dependence on the spatial location of controllers and sensors. Section 3 comprises a brief general review of the more recent OCL and OSL literature, emphasizing the "final" problem (i.e. system identification, state estimation, or optimal control) for which they were developed. A classification of methods is presented in section 4, where the methods reviewed in section 3 are compared according to their main structural characteristics. The paper ends with some comments and concluding remarks in section 5.

2. MODELLING PRELIMINARIES

The purpose of this section is to present a brief summary on the following topics: (1) linear models for DPS, (2) equivalent model description, (3) N-modal approximation, and (4) model dependence on the spatial location of controllers and sensors. The motivation for this will become clear in section 3 when the several OSL and OCL methods will be reviewed. As it will be emphasized in section 4, the major factor for classifying the OSL and OCL methods will rely on when optimization techniques are applied, either before or after considering any model approximation; and the so-called N-modal is the most used approximation technique in the OSL and OCL literature.

Notation

The notation used in this section is summarized as follows:

$\mathbb{R}^n$	: n-dimentional Euclidean space.
$\partial\Omega$	: Boundary of $\Omega \subset \mathbb{R}^n$.
$<,>_H$	: Inner product in a Hilbert space H.
$\|\| \ \|\|_H$	: Norm in a Hilbert space H.
D(L)	: Domain of a transformation L.
L*	: Adjoint of a transformation L.
$\dot{w}$	: Time derivative of w ($\dot{w} = \partial w/\partial t$).
E	: The expectation operator, as usual.
$M[\mathbb{R}^k,\mathbb{R}^\ell]$	: Linear space of all real matrices ℓ by k ($M[\mathbb{R}^k] = M[\mathbb{R}^k,\mathbb{R}^k]$).
$B\ell t[X,Y]$	: Normed linear space of all bounded linear transformations of X into Y, X and Y being normed linear spaces ($B\ell t[X]=B\ell t[X,X]$).

The real spaces ℓ_2, $L_2(0,T)$, $L_2(\Omega)$, $L_2(0,T;H)$, $C(0,T)$, $C(\Omega)$, $C^2(\Omega)$, and $C(0,T;H)$ will have their standard meanings (e.g., see [31]).

A linear model for DPS in $L_2(\Omega)$

Technical details are omitted throughout this section and the reader is here, once and for all, referred to the available literature. As far as Hilbert space methods are concerned see, for instance, Naylor and Sell [58] or Weidmann [77], among others. Classical references for the semigroup theory are Hille and Phillips [40] and Yosida [78]. For an introduction to semigroups towards control theory see, for example, Balakrishnam [12] or Curtain and Pritchard [31,32].

Let U (the input or control space), H (the state space), and V (the observation or output space) be Hilbert spaces, and consider a linear dynamical system modelled by an autonomous inhomogeneous abstract differential equation as follows

$$\dot{y} = A\,y + B\,u \quad , \quad y(0) = y_o \in H \,, \tag{1}$$

where $u \in L_2(0,T;U)$, $B \in B\ell t[U,H]$, and the (closed linear, but possibly unbounded) operator A: $D(A)\to H$ is the infinitesimal generator of a strongly continuous semigroup $\{T_t \in B\ell t[H];t\geq 0\}$, where the domain $D(A)$ is dense in H. The mild solution of (1) is given by

$$y(t) = T_t y_o + \int_0^t T_{t-s} B\,u(s)ds \,, \tag{2}$$

with $y \in C(0,T;H)$. Furthermore, let v, $z \in L_2(0,T;V)$ and $C \in B\ell t[H,V]$, and suppose the state y is observed according to the following measurement equation

$$z = C\,y + v \quad . \tag{3}$$

Now set $H=L_2(\Omega)$, Ω being a simply connected open set in $\mathbb{R}^n$, and consider a linear time-invariant DPS governed by a parabolic PDE as in (1). For example suppose a special case where the system operator A is a second-order elliptic self-adjoint one of the form

$$A = \sum_{\nu} \alpha_\nu \mathcal{D}^\nu \quad , \tag{4}$$

with

$$\nu=(\nu_1,\ldots,\nu_n)\in I^n \;;\quad \nu_q \in I=\{0,1,2\} \,,$$

$$\forall q=1,2,\ldots,n \,,$$

$$|\nu|=\sum_{q=1}^n \nu_q \text{ is such that } 0 \leq |\nu| \leq 2 \,,$$

$$\alpha_\nu \in C^2(\Omega) \quad ,$$

$$\mathcal{D}^\nu = \frac{\partial^{|\nu|}}{\partial x_1^{\nu_1} \ldots x_n^{\nu_n}} : H^2(\Omega)\to L_2(\Omega) \quad ,$$

$$H^2(\Omega)=\{w \in L_2(\Omega)\colon \mathcal{D}^\nu w \in L_2(\Omega);\ |\nu|=1,2\},$$

$$D(A)=\{w \in H^2(\Omega)\colon Lw=0 \quad \text{on } \partial\Omega\} \,,$$

where L denotes a linear operator defined on $\partial\Omega$ (standing for the boundary conditions). Moreover, assume that there exists an infinite divergent real sequence $\{\lambda_i;i=1,2,\ldots\}$ of eigenvalues of A, which is bounded above and non-increasingly ordered; that is

$$A\,\phi_i = \lambda_i\phi_i \quad ,$$

$$\lambda_{i+1} \leq \lambda_i < \lambda_o < \infty \quad ,$$

$$|\lambda_i| \to \infty \quad \text{as} \quad i \to \infty \quad ,$$

where $\{\phi_i \in D(A);i=1,2,\ldots\}$ is an orthonormal basis for $L_2(\Omega)$ of eigenvectors of A, such that the solution in (2) has an unique Fourier series expansion

$$y(t) = \sum_{i=1}^{\infty} a^i(t)\phi_i \quad , \qquad (5)$$

yielding, for $y(t) \in D(A)$,

$$A\, y(t) = \sum_{i=1}^{\infty} \lambda_i a^i(t)\phi_i \quad , \qquad (6)$$

where $a^i \in C(0,T)$ is given by

$$a^i(t) = \langle y(t),\phi_i\rangle_{L_2(\Omega)} \quad ,$$

and the semigroup $\{T_t \in B\ell t[L_2(\Omega)];t\geq 0\}$ generated by A is

$$T_t y(s) = \sum_{i=1}^{\infty} e^{\lambda_i t}\, a^i(s)\phi_i \quad .$$

Now set $U = \mathbb{R}^p$, and assume that the input transformation $B \in B\ell t[\mathbb{R}^p, L_2(\Omega)]$ is such that

$$Bu(t) = \sum_{j=1}^{p} \beta_j u_j(t) \quad ,$$

with $\beta_j \in L_2(\Omega)$ and $u_j \in L_2(0,T)$ for each $j=1,2,\ldots,p$. By the Fourier series expansion of β_j one gets

$$B\, u(t) = \sum_{i=1}^{\infty} \langle u(t),b_i\rangle_{\mathbb{R}^p}\, \phi_i \quad , \qquad (7)$$

where

$$u = (u_1,\ldots,u_p) \in L_2(0,T;\mathbb{R}^p) \quad ,$$

$$b_i = (\langle\beta_1,\phi_i\rangle_{L_2(\Omega)},\ldots,\langle\beta_p,\phi_i\rangle_{L_2(\Omega)}) \quad .$$

Finally set $V = \mathbb{R}^m$, and let the output transformation $C \in B\ell t[L_2(\Omega), \mathbb{R}^m]$ be given by

$$C\, y(t)=(\langle y(t),\gamma_1\rangle_{L_2(\Omega)},\ldots,\langle y(t),\gamma_m\rangle_{L_2(\Omega)}), \qquad (8)$$

where $\gamma_k \in L_2(\Omega)$ for each $k=1,2,\ldots,m$, such that

$$z_k(t) = \langle y(t),\gamma_k\rangle_{L_2(\Omega)} + v_k(t) \quad ,$$

with $z=(z_1,\ldots,z_m)$, $v=(v_1,\ldots,v_m) \in L_2(0,T;\mathbb{R}^m)$.

An equivalent model in ℓ_2

The equations (5)-(7) supply an equivalent representation in ℓ_2 for the system model (1), (4) in $L_2(\Omega)$, as follows:

$$\dot{a} = A\, a + B\, u \quad , \quad a(0) \in \ell_2 \quad , \qquad (9)$$

where $B \in B\ell t[\mathbb{R}^p, \ell_2]$ is such that

$$Bu(t)=(\langle Bu(t),\phi_1\rangle_{L_2(\Omega)},\langle Bu(t),\phi_2\rangle_{L_2(\Omega)},\ldots)$$

$$=(\langle u(t),b_1\rangle_{\mathbb{R}^p},\langle u(t),b_2\rangle_{\mathbb{R}^p},\ldots) \quad ,$$

and $A{:}D(A)\to\ell_2$ is a closed densely defined linear operator

$$Aa(t)=(\langle Ay(t),\phi_1\rangle_{L_2(\Omega)},\langle Ay(t),\phi_2\rangle_{L_2(\Omega)},\ldots)$$

$$=(\lambda_1 a^1(t),\lambda_2 a^2(t),\ldots) \quad ,$$

$$D(A)=\{\omega=(\omega_1,\omega_2,\ldots)\in \ell_2 : \sum_{i=1}^{\infty}|\lambda_i\omega_i|^2<\infty\} \quad ,$$

generating a strongly continuous semigroup $\{T_t \in B\ell t[\ell_2];t\geq 0\}$

$$T_t a(s)=(\langle T_t y(s),\phi_1\rangle_{L_2(\Omega)},\langle T_t y(s),\phi_2\rangle_{L_2(\Omega)},\ldots)$$

$$=(e^{\lambda_1 t}a^1(s),e^{\lambda_2 t}a^2(s),\ldots) \quad .$$

The mild solution of (9) is then given by

$$a(t) = T_t a(0) + \int_0^t T_{t-s}\, B\, u(s)ds \quad ,$$

with $a=(a^1,a^2,\ldots)\in C(0,T;\ell_2)$. By the Fourier expansion of γ_k in (8) one gets the following equivalent representation in ℓ_2 for the measurement equation (3) in $L_2(\Omega)$.

$$z = C\, a + v \quad , \qquad (10)$$

where $C \in B\ell t[\ell_2,\mathbb{R}^m]$ is given by

$$C\, a(t)=(\langle a(t),c_1\rangle_{\ell_2},\ldots,\langle a(t),c_m\rangle_{\ell_2}) \quad ,$$

with

$$c_k=(\langle\gamma_k,\phi_1\rangle_{L_2(\Omega)},\langle\gamma_k,\phi_2\rangle_{L_2(\Omega)},\ldots) \quad ,$$

for each $k=1,2,\ldots,m$.

An approximate model in $\mathbb{R}^N$

The so-called N-modal approximation consists in truncating the Fourier series expansions involved in their first N terms, yielding a Galerkin-like approximation for the state y in (5),

$$y_N(t) = \sum_{i=1}^{N} a^i(t)\phi_i \quad ,$$

$y_N \in C(0,T;H_N)$, where H_N is the N-dimensional linear subspace of $L_2(\Omega)$ spanned by $\{\phi_i;\ i=1,2,\ldots,N\}$. This supplies an approximate representation in $\mathbb{R}^N$ for the equivalent system model (9) in ℓ_2, given by

$$\dot{a}_N = A_N\, a_N + B_N\, u \quad , \quad a_N(0) \in \mathbb{R}^N \quad , \qquad (11)$$

where

$$A_N = \mathrm{diag}(\lambda_1,\ldots,\lambda_N) \in M[\mathbb{R}^N] ,$$

$$B_N = [b_1,\ldots,b_N]^* \in M[\mathbb{R}^p,\mathbb{R}^N] ,$$

whose solution is

$$a_N(t) = T_t^N\, a_N(0) + \int_0^t T_{t-s}^N\, B_N u(s)ds ,$$

with $a_N = (a^1,\ldots,a^N) \in C(0,T;\mathbb{R}^N)$, and

$$T_t^N = e^{A_N t} = \mathrm{diag}(e^{\lambda_1 t},\ldots,e^{\lambda_N t}) \in M[\mathbb{R}^N];\ t>0.$$

The observation $z_N = (z_1^N,\ldots z_m^N) \in L_2(0,T;\mathbb{R}^m)$ is given by the following approximate version of (10)

$$z_N = C_N\, a_N + v , \tag{12}$$

with

$$C_N = [c_1^N,\ldots,c_m^N]^* \in M[\mathbb{R}^N,\mathbb{R}^m] ,$$

$$c_k^N = (\langle\gamma_k,\phi_1\rangle_{L_2(\Omega)},\ldots,\langle\gamma_k,\phi_N\rangle_{L_2(\Omega)})$$

for each $k=1,\ldots,m$.

Controllers and sensors spatial location dependence

Suppose the input transformation B in (7) depends on a vector $x^c = (x_1^c,\ldots x_p^c) \in \mathbb{R}^{np}$ as follows. Let the input (or control) coefficients depend on x^c in the following way.

$$\beta_j = \beta_{x_j^c} , \quad x_j^c \in \Omega \subset \mathbb{R}^n ,$$

where x^c describes the controllers spatial location, such that

$$[B\,u(t)](x) = \sum_{j=1}^p \beta_{x_j^c}(x)\; u_j(t) .$$

For instance, let

$$0 < \varepsilon < \inf_{1\le j\le p}\ \inf_{x\in\partial\Omega} \| x_j^c - x \|_{\mathbb{R}^n} ,$$

such that the chosed ball $\sigma_\varepsilon[x_j^c]$ of radius ε centered at x_j^c is contained in Ω for each $j=1,\ldots,p$, and let $\mu_\varepsilon>0$ be the usual measure of $\sigma[x_j^c]$. Now set

$$\beta_{x_j^c}(x) = \begin{cases} \mu_\varepsilon^{-1} , & \text{if } x \in \sigma_\varepsilon[x_j^c] , \\ 0 , & \text{otherwise} . \end{cases}$$

Hence

$$\langle\beta_j,\phi_i\rangle_{L_2(\Omega)} = \mu_\varepsilon^{-1} \int_{\sigma_\varepsilon[x_j^c]} \phi_i(x)dx .$$

Therefore the approximate (N-modal) representation for the input transformation in (11) is given by

$$B_N = \mu_\varepsilon^{-1} \begin{bmatrix} \int_{\sigma_\varepsilon[x_1^c]} \phi_1(x)dx & \cdots & \int_{\sigma_\varepsilon[x_p^c]} \phi_1(x)dx \\ \vdots & & \vdots \\ \int_{\sigma_\varepsilon[x_1^c]} \phi_N(x)dx & \cdots & \int_{\sigma_\varepsilon[x_p^c]} \phi_N(x)dx \end{bmatrix}$$

$$B_N = B_N(x^c) \in M[\mathbb{R}^p,\mathbb{R}^N] .$$

In a similar fashion, suppose the output transformation C in (8) depends on a vector $x^s = (x^s \ldots\ldots x_m^s) \in \mathbb{R}^{nm}$ as follows. Let the output coefficients depend on x^s in the following way.

$$\gamma_k = \gamma_{x_k^s} , \quad x_k^s \in \Omega \subset \mathbb{R}^n ,$$

where x^s describes the sensors spatial location, such that

$$z_k(t) = \langle y(t),\gamma_{x_k^s}\rangle_{L_2(\Omega)} + v_k(t) .$$

For instance, set

$$\gamma_{x_k^s}(x) = \begin{cases} \mu_\varepsilon^{-1} , & \text{if } x \in \sigma_\varepsilon[x_k^s] , \\ 0 , & \text{otherwise} , \end{cases}$$

where $\sigma_\varepsilon[x_k^s]$ is defined as $\sigma_\varepsilon[x_j^c]$, with x^c replaced by x^s. Therefore the approximate (N-modal) representation for the outuput transformation in (12) is given by

$$C_N = \mu_\varepsilon^{-1} \begin{bmatrix} \int_{\sigma_\varepsilon[x_1^s]} \phi_1(x)dx & \cdots & \int_{\sigma_\varepsilon[x_1^s]} \phi_N(x)dx \\ \vdots & & \vdots \\ \int_{\sigma_\varepsilon[x_m^s]} \phi_1(x)dx & \cdots & \int_{\sigma_\varepsilon[x_m^s]} \phi_N(x)dx \end{bmatrix}$$

$$C_N = C_N(x^s) \in M[\mathbb{R}^N;\mathbb{R}^m] .$$

Before closing this section it is worth remarking on pointwise controllers and sensors. Consider a formal approach by letting $\varepsilon\to 0$. In such a case the input (or control) and output coefficients $\beta_{x_j^c}$ and γ_x can be thought of as Dirac measures, that i

$$\beta_{x_j^c}(x) = \delta(x-x_j^c) ,$$

$$\gamma_{x_k^s}(x) = \delta(x-x_k^s) \quad ,$$

thus supplying approximate representations for the input and output transformations of the following form.

$$B_N(x^c) = \begin{bmatrix} \phi_1(x_1^c) & \cdots & \phi_1(x_p^c) \\ \vdots & & \vdots \\ \phi_N(x_1^c) & \cdots & \phi_N(x_p^c) \end{bmatrix} ,$$

$$C_N(x^s) = \begin{bmatrix} \phi_1(x_1^s) & \cdots & \phi_N(x_1^s) \\ \vdots & & \vdots \\ \phi_1(x_m^s) & \cdots & \phi_N(x_m^s) \end{bmatrix} .$$

However, the above formal approach leads to unbounded transformations, and $L_2(\Omega)$ is not an appropriate state space anymore.

An illustrative example of an OCL and OSL problem

For simplicity consider a stochastic version of the approximated model in (11), (12).

$$da_N(t)=A_N\, a_N(t)dt+B_N(x^c)[u(t)dt+dw(t)] \quad ,$$

$$dz_N(t)=C_N(x^s)a_N(t)dt+dv(t) \quad .$$

Here $\{w(t);\ t\geq 0\}$ and $\{v(t);\ t\geq 0\}$ are independent Wiener processes in $\mathbb{R}^p$ and $\mathbb{R}^m$ with incremental covariance matrices $R_w \in M[\mathbb{R}^p]$ and $R_v \in M[\mathbb{R}^m]$, standing for input disturbance and observation noise, respectively. $\{u(t);\ 0\leq t\leq T\}$ is an $\mathbb{R}^p$-valued second-order stochastic control, that is

$$E\{\|u\|^2_{L^2(0,T;\mathbb{R}^p)}\} = E\{\int_0^T \|u(t)\|^2_{\mathbb{R}^p} dt\} < \infty ,$$

which depends only on the past observations $\{z_N(\tau);\ 0\leq\tau\leq t\}$; and $a_N(0)$ is a zero mean Gaussian random variable in $\mathbb{R}^n$ with covariance matrix $P_o \in M[\mathbb{R}^N]$, which is independent of $w(t)$ and $v(t)$. A simplified version for the linear quadratic Gaussian (LQG) problem is to find a stochastic control u, as above, which minimizes the cost

$$J(u) = E\{\|a_N(T)\|^2_{\mathbb{R}^N}\}+E\{\|a_N\|^2_{L_2(0,T;\mathbb{R}^N)}\}+$$
$$+ E\{\|u\|^2_{L_2(0,T;\mathbb{R}^p)}\} \quad ,$$

where the first two criteria characterize the accuracy in which the state can be driven to zero at the final time and along the whole trajectory, respectively, and the third one stands for the control energy. For simplicity it has been assumed identity weighting matrices for each criterion. According to the separation principle, the solution $u_N = u_N(x^c,x^s)$ is given by

$$u_N(t) = -B_N^*(x^c)Q_N(t)\hat{a}_N(t) \quad ,$$

where the symmetric feedback control matrix $Q_N(t)$ in $M[\mathbb{R}^N]$ is the unique solution of the backwards Riccati equation

$$\dot{Q}_N(t) = Q_N(t)B_N(x^c)B_N^*(x^c)Q_N(t)-$$
$$-Q_N(t)A_N-A_N^*Q_N(t)-I_N \ , \ Q_N(T)=I_N \ ,$$

and $\hat{a}_N(t)$ denotes the Kalman-Bucy filtered estimate of the state $a_N(t)$,

$$d\hat{a}_N(t)=[A_N-P_N(t)C_N^*(x^s)R_v^{-1}C_N(x^s)-$$
$$-B_N(x^c)B_N^*(x^c)Q_N(t)]\hat{a}_N(t)dt+$$
$$+P_N(t)C_N^*(x^s)R_v^{-1}dz_N(t) \ , \quad \hat{a}_N(0)=0 \ ,$$

where the error covariance $P_N(t)=E\{[a_N(t)-\hat{a}_N(t)][a_N(t)-\hat{a}_N(t)]^*\}$ in $M[\mathbb{R}^N]$ is the unique solution of the Riccati equation

$$\dot{P}_N(t) = A_NP_N(t)+P_N(t)A_N^*+B_N^*(x^c)R_wB_N(x^c)-$$
$$-P_N(t)C_N^*(x^s)R_v^{-1}C_N(x^s)P_N(t) \ , \ P_N(0)=P_o \ .$$

The optimal cost is then given by

$$J[u_N(x^c,x^s)]= \text{trace } P_N(T)+\int_0^T \text{trace } P_N(t)dt+$$
$$+\int_0^T \text{trace } Q_N(t)P_N(t)C_N^*(x^s)R_v^{-1}C_N(x^s)P_N(t)dt \ .$$

An example of an OCL and OSL problem (for a fixed number of sensors and controllers) is to sellect $(x^c,x^s) \in \Omega^p \times \Omega^m \subset \mathbb{R}^{np} \times \mathbb{R}^{nm}$ which minimizes the cost $J[u_N(x^c,x^s)]$ of the above-described optimal (closed-loop) control strategy.

3. A GENERAL REVIEW

In this section we present a brief review of the more recent OSL and/or OCL methods considered in the current literature. The bibliography mentioned here comprises over 40 widely available papers published in the last decade. The several contributions in the field are primary grouped according to the main goal for which the OSL and OCL problems are developed (instead of using a chronological order), namely: System Identification, State Estimation and Optimal Control. Since the main goal behind an OSL problem may be any of the above-mentioned, the following further abbreviations concerning the methods dealing with OSL will be adopted:

SLI: OSL for System Identification.
SLE: OSL for State Estimation.
SCL: OSL and OCL for Optimal Control.

SLI: (Optimal Sensors Location for System Identification)

A few papers have appeared on the SLI problem for DPS, and they represent rather different approaches. Therefore it seems that an individual analysis is the most suitable way for reviewing them. For a previous discussion on this topic, based mainly on observability arguments, see Goodson and Polis [37].

Le Pourhiet and Le Letty [49] proposed two algorithms, somewhat similar to each other, as an SLI procedure for deterministic DPS. The basic idea was to maximize, at each iteration, the identification error sensivity (according to pre-established identifiability definitions) with respect to the location of a new sensor. The first algorithm concerns the improvement in the sensitivity criterion by adding a new sensor to the set of all sensors already located in previous iteractions; and the second one also takes into account the location of the new sensor at the preceding iteraction. Both algorithms stop when the placement of a new sensor no more adds any substantial improvement as far as the identification error sensitivity is concerned. It is worth emphasizing that in the above described approach it was not assumed an "a priori" fixed number of available sensors.

Sokollik[73] considered both the number and location of sensors, as well as the measurement times, for identifying DPS. The distribuited model was approximated by a lumped one by using finite-differences. In this way both the time and space domains were discretized with constant sampling rates. The optimal space-time net (i.e., the optimization of time and spatial location for the measurements) was given by minimizing the parameter estimate covariance, which was performed by stochastic approximation schemes presented in [71,72].

Qureshi, Ng and Goodwin [63] presented a method to design optimal experiments for identifying DPS through noisy observations. Besides the SLI, it was also considered the determination of boundary perturbations for identifying not necessary linear systems. The optimization criterion to be maximized was the determinant of the Fisher's information matrix associated to the parameters to be identified, which depends on both the boundary perturbations and spatial location of the observation points. The design method was developed for hyperbolic and parabolic PDE.(1)

(1) The SLI problem was also recently considered by Carotenuto and Raiconi [80], and Rafajłowicz [81].

SLE: (Optimal Sensors Location for State Estimation)

Several papers dealing with the SLE problem have already appeared (cf. Tzafestas [75]), and they present some common characteristics. For instance, every method discussed here considers white Gaussian observation noise when dealing with the (stochastic) filtering problem. Aidarous, Gevers and Installé [2,4] are the only to consider discrete-time observation process. Cannon and Klein [22] and Caravani, Di Pillo and Grippo [23] consider a dynamical equation without input disturbance, while the other always assume Gaussian input disturbances. The main characteristic of the majority of the SLE methods analysed here is the reduction of an infinite-dimensional system to a finite-dimensional one, by truncating the(infinite) Fourier expansion of either the state or the estimates in its first N terms (N-modal approximation), according to the increasing order of the partial differential operator eigenvalues. In this way the filtering procedure is applied either in a finite-dimensional state space or in a infinite-dimensional one, respectively. Concerning the latter case, when the state estimate error covariance appears explicitly in the performance index, such an approximation is applied on the covariance operator rather than on the estimate itself. In the light of the above introductory discussion, the SLE bibliography reviewed here can be gathered in two major groups.

GROUP I:

Yu and Seinfeld [79], Caravani, Di Pillo and Grippo [23], Omatu, Koide and Soeda [59], and Sawaragi, Soeda and Omatu [69] treated the SLE problem in a somewhat similar fashion. The idea behind the approach used was to represent the state variable $y(x,t)$ as a infinite series of eigenfunctions of the partial differential operator modelling the DPS. This yields an equivalent model described by an ODE in the sequence $a(t)$, comprising the coefficients of that expansion. Such an infinite sequence is approximated by an N-dimensional vector $a_N(t)$, obtained by truncating it in its first N terms. This supplies the state N-modal approximation $y_N(x,t)$ (cf. section 2). The state estimation problem is then approached by determining the finite-dimensional estimate $\hat{a}_N(t)$ for the N-modal approximation estimate $\hat{y}_N(x,t)$. The SLE x^s is finally determined through $\hat{y}_N(x,t)$ by optimizing some appropriate criterion (cf. Fig. 1).

In [23] it was investigated the location of a single sensor for estimating the initial state in the one-dimensional heat equation. It was assumed homogeneous boundary conditions in the state, such that the DPS was excited only by the unknown initial condition. The noisy sensor placement was performed by minimizing the maximum mean square error for the initial state.

The SLE was accomplished in [79,59,69] by minimizing the trace of the estimate error covariance matrix at the final time. A recursive algorithm was proposed in [79], which determines the optimal location of one sensor in terms of the previously located sensors.

In [59,69] it was presented existence theorems concerning the solution of the SLE problem in infinite dimension. Theorems establishing necessary and sufficient conditions for the SLE, before considering any state-space approximation, were also presented.

GROUP II:

The methods discussed above (Group I) used (either implicitly or explicitly an N-modal approximation for the state $y(x,t)$, and so they applied finite-dimensional filtering algorithms to $a_N(t)$. On the other hand, Bensoussan [15], Aidarous, Gevers and Installé [2,4], Amouroux, Babary and Malandrakis [10], Kumar and Seinfeld [47], Curtain and Ichikawa [30] and Nakamori, Miyamoto, Ikeda and Sawaragi [57] used a different approach. The idea behind this was to apply infinite-dimensional filtering to the state $y(x,t)$, and then to represent the state estimate $\hat{y}(x,t)$ as a infinite series with the coeficients sequence $\hat{a}(t)$; which is truncated in its first N terms yielding the vector $\hat{a}_N(t)$ of the estimate N-modal approximation $\hat{y}_N(x,t)$.

In such methods this approximation procedure was actually applied only on the covariance operator, rather than in the state estimate itself. The great majority of the above-mentioned papers faced the SLE problem by minimizing a cost function given in terms of the trace of the N-modal approximation for the estimate error covariance operator; thus supplying the SLE x^s (cf. Fig. 1).

A theoretical treatment for the SLE problem was proposed in [15] by using functional analysis techniques based on the Lions'[51] approach to control theory for DPS. The existence of solutions for the SLE problem, as well as necessary conditions for optimality, were established. This was achieved by formulating the SLE problem as an optimal control one on the Riccati equation describing the evolution of the estimate error covariance operator.

In [2] it was initially considered the location of a single sensor, and the procedure was then extended to cover the case of several sensors. They assumed discrete-time observations. The SLE problem was approached by minimizing the spatial integral of the N-modal approximation for the estimate error covariance. In [4] they proved the existence of solutions for the SLE problem, and also the location algorithm convergence, for the method presented in [2].

It was used in [10] a weighting function for the terms in the trace of the error covariance N-modal approximation. This was done in order to increase the accuracy for the first coefficients of the state N-modal approximation.

In [47] the computational problem concerning the minimization of the integral of the trace of the estimate error covariance matrix was overcome. They replaced that matrix by an upper bound of it, given in terms of the covariance matrix associated to the free system. It was also analysed the SLE problem sensitivity with relation to boundary condition, observation noise covariance, and initial error covariance variations.

The filtering problem was approached in [30] by using abstract evolution equations in Hilbert space. As in [15] the SLE problem was rigorously treated as an optimal control one, where the control variable characterizes the sensor location. Opposite to [15], they used the mild evolution operators approach for considering existence theorems for SLE, as well as necessary conditions for optimality.

In [57], as in [15, 30], the SLE problem was approached as a deterministic optimal control one, whose basic cost function was given by the trace of the estimate error covariance operator and by a further term standing for the control cost. Semigroup theory was used as in [30]. It was established an existence theorem and sufficient conditions for optimality, by using a sensitivity criterion given by the trace of the information operator; which can be thought of as an extension of the Fisher's information matrix to infinite-dimensional spaces. The computational effort in connection to the above criterion was claimed to be smaller compared with that required for the trace of the filter covariance. For implementation, it was suggested an N-modal approximation for that information operator.

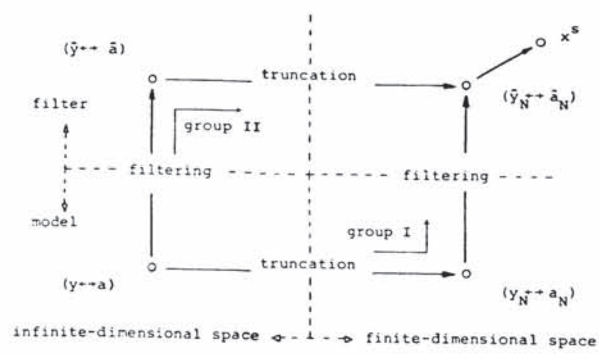

Fig. 1: N-modal approximations for SLE.

GROUP III:

Cannon and Klein [21,22], Klein [44], Ewing and Higgins [34], Chen and Seinfeld [25], Kumar and Seinfeld [48], Morari and O'Dowd [56], also investigated the SLE by considering the estimation problem in a infinite-dimensional space. However, in each one of the above papers it was presented a somewhat specific characteristic, which suggests a separate review rather than an inclusion in the previous groups.

The heat equation in one-dimensional spatial domain and without forcing term was considered in [21,22]. Although the DPS was supposed to operate in a deterministic environment, it was allowed uncertanties in the initial and boundary conditions, as well as in the observation process. The location of a single transducer, which was assumed to average the measurements over a small neighborhood in the spatial domain,was investigated. The theory behind the method applied analytical arguments for establishing an (upper bound) estimate for the state, which was used for supplying estimates for the error between the state itself and numerical approximations of it. The SLE was then accomplished by minimizing these error estimates. The same approach was also considered in [44].

In [25] the optimality criterion was given by the space-time integral of the trace of the estimate error covariance. The spatial domain was "a priori" discretized in order to avoid a possible sensors clustering in a small region. The SLE problem was then approached as an optimal control one in which: (1) the state dynamics is given by the matrix PDE describing the estimate error covariance evolution, and (2) the control variables are characterized by a Boolean vector indicating either the presence or absence of sensors over the discrete spatial domain. Although it was not considered a finite-dimensional approximation for the state-space, the algorithm developed for sensors location requires at each iteration the resolution of two matrix PDE (the covariance evolution and its adjoint), whose solution method was not discussed in [25]. This method has been applied in [48] for state estimation in tubular chemical reactors, where they have used orthogonal collocation techniques for reducing the infinite-dimensional system to a finite-dimensional one.

The SLE problem was investigated in [56] by assuming that the DPS is driven by nonstationary input disturbances. Their approach was based on the G-K (Goodson-Klein) observability criterion, whose conditions may not be satisfied for a limited number of sensors. In this way it was proposed an SLE method by minimizing the information loss associated to the nonobservable subspace. This optimality criterion is given by the spatial integral of the trace of the estimate error covariance operator. Such an error is mainly caused by the lack of observability due to the presence of nonstationary input disturbances. Although the theory was not developed in a finite-dimensional space, they used state approximations for examples implementation, including experimental results.

OCL: (Optimal Controllers Location)

The OCL problem has been investigated mainly by the French School. Lions [52], Amouroux [5],Amouroux and Babary [6-9], Aidarous [1], Aidarous, Gevers, and Installé [3], and Burger [17,18] considered the optimal location of control points (actuators) for DPS. Generally these methods presented several common characteristics, applied to somewhat similar mathematical models. For instance, those which considered feedback control assumed that the observation points (i.e., the sensors location) were "a priori" determined, as opposite to the SCL methods discussed latter in this section.

Like the SLE methods in Group I, the idea behind the OCL approach used in [1,3,5-9] lies on truncating the coefficients sequence $a(t)$ of the eigenfunction series expansion for the state $y(x,t)$. After that, the optimal control strategy is determined for the system N-modal approximation. In this way the optimal control problem, for the state N-modal approximation $y_N(x,t)$, is approached in a finite-dimensional state space in terms of the truncated vector $a_N(t)$. The same approximation technique was also used in [17,18], where the OCL problem was faced from a rather different point of view.

The results presented in [5,6] are related to the concepts of controllability and reachability, rather than to the optimal control problem. The main goal was to maximize, over all possible control points, the volume of a hyperellipsoid in the state space comprising the reachable states for bounded pointwise controls. The OCL was formulated according to the necessary and sufficient conditions for reachability of the truncated system.

The optimality criterion in [8,9] was given by the overall control energy and by the state accuracy at the final time. They also reviewed other two OCL procedures besides the state truncation one: an iterative method using gradient-like algorithms, and a parametrization method using N-modal approximation for the distributed control. The pointwise controllers considered in [8] were extended to the case of zones of action in [9], where both approaches were compared. In [7] they considered several performance criteria for the optimal pointwise control problem. For instance, the minimization of the truncated optimal control and the final state error norms, were discussed among others.

Opposite to [5-9], the approach in [3]

considered discrete-time observations and stochastic (Gaussian) input disturbances. As far as the optimality criterion is concerned, [3] minimized a mixed cost function comprising the overall control energy and state accuracy along the whole trajectory. An interesting analysis on the duality between the SLE and OCL problems, according to [2] and [3], was also presented in [3].

In [18] the OCL was discussed form both points of view: zones of action and pointwise controllers. After using a state N-modal approximation, it was then assumed that the system is static (rather than dynamical), thus referring the OCL problem to an ODE model approximated by an algebraic equation. A geometrical approach was considered, by using orthogonal projection arguments, for minimizing the distance between the desired "state" (for the static system) and the reachability linear subspace.

Theoretical aspects regarding the OCL for pointwise controls was slightly investigated in chapter 4 of [52] for deterministic DPS. There a rigorous abstract approach for establishing the existence of optimal position for Dirac measures was considered.

SCL: (Optimal Sensors and Controllers Location)

The SCL problem refers to the optimal location of both sensors and controllers, generally for closed-loop optimal control problems in DPS. In case of feedback control, such a combined procedure involving OSL and OCL problems may eventually concern state estimation as well (and hence SLE as a special case of OSL).

Such a problem has been investigated by Amouroux, Di Pillo and Grippo [11], El Jai [33], Ichicawa and Ryan [41,42], Courdesses [27], and Malandrakis [54]. Instead of the author by author review procedure used so far, it seems more appropriate to review the above SCL literature according to the main characteristics used to face the problem. This is motivated by the several common points shared by the subsets of the above-mentioned set of papers.

Concerning the environment in which the DPS is supposed to envolve, [54,41,42] considered (Gaussian) disturbances corrupting the control action, although all the papers up to [27] assumed observation (Gaussian) noise corrupting the measurements. A completely deterministic formulation was considered in [27]. In every of the above-mentioned papers it was assumed continuous time operation; and [33] was the only one to consider open-loop control and a variable number of sensors and controllers. Pointwise controls were assumed in [54,27].

For the SCL methods applied to stochastic DPS in [11,54,41,42] the optimal control strategy was given according to the separation principle, after performing the state estimation. The stochastic regulator problem for evolution equations was considered in [41,42] by using a semigroup approach. In [11] the filtering procedure was applied in finite-dimensional spaces by considering a state N-modal approximation. On the other hand, [54,41,42] applied infinite-dimensional filtering and used N-modal approximation for operators associated to the LQG (Linear-Quadratic-Gaussian) optimal control problem (i.e., they used N-modal approximation for the feedback and estimate error covariance operators). In a similar fashion, the deterministic approach considered in [27] involved N-modal approximation in connection to the linear-quadratic (deterministic) optimal control problem. In the open-loop approach presented in [33] the pointwise OSL was implemented for estimating the initial state, and the OCL for reaching a desired final state using minimum energy controls.

The optimality criterion involved in the method presented in [11] was given by the minimization of the state estimate error at the final time and the overall control energy. Several possible criteria and practical considerations for the SCL problem, including the sensors and controllers number optimization, were discussed in [33]. In [54,41,42,27] the cost functional to be minimized comprised three terms: final state accuracy, state accuracy along the whole trajectory, and the overall control energy. The existence of an optimal location was established in [41,42], where it was also presented a comparative analysis involving either separate or simultaneous location of sensors and controllers.

The OSL problem for deterministic closed-loop control was also considered by Koivo and Kruch [45]. Such an approach, which was one of the first to appear, was quite different from those described above, since the control was supposed to act only on a fixed boundary point. Therefore this characterized an OSL problem for closed-loop control, rather than an SCL problem. Some theoretical aspects regarding the existence of solutions for a particular OSL problem in feedback control for deterministic DPS were presented by Lions [52]. As in [45], the OSL problem for closed-loop control of temperature distribution was also considered by Kaizer [43]. Further applications involving the SCL problem for DPS were investigated by Lee, Koppel and Lim [50] as well.

4. A CLASSIFICATION OF METHODS

As it has been commented on before (eg. see [46,61,67]), sooner or later one will be faced with approximation techniques (either for modelling or numerical and physical implementation) when dealing with any problem in DPS. For the OCL and OSL problems, it can be observed from the previous section that the N-modal approximation (also called truncation of eigenfunctions - or Fourier, or harmonic - expansion, as an approximation scheme resulting from the separation of variables technique) is certainly the most used for sensors and controllers location in DPS.

The purpose of this section is twofold. First of all some relevant characteristics of those methods for OCL and OSL in (dynamical) DPS which use, in one way or another, N-modal approximation schemes are summarized. Such methods are then classified according to the stage of the optimization procedure in which N-modal approximations are used.

Methods characteristics

Table 1 displays some models and methods characteristics for that part of the literature reviewed in the preceding section which uses N-modal approximation. The following notation has been adopted in table 1, where the first four items concern the external action in the DPS.

1. INPUT: The input (or forcing term) in the dynamic equation can be described either by stochastic disturbance (w) and/or stochastic control (u), or by deterministic control (u_d). Null input is denoted by (0).

2. OBSERVATION NOISE: The presence or absence of noise corrupting the measurements will be denoted by either (v) or (0), respectively.

3. BOUNDARY CONDITIONS (BC): They can be either homogeneous (H), or inhomogeneous; whose external action in the boundary can be described either by stochastic (S) or by deterministic (D) processes.

4. INITIAL CONDITIONS (IC): Both known and unknown IC will be denoted by (D) or (S), whenever they are given by deterministic or stochastic processes, respectively. When an unknown IC is to be estimated, it will be denoted by (E). Null IC are represented by (0).

5. NUMBER OF LOCATED POINTS AT SIMULATED EXAMPLES (#): For SCL methods, the first number displayed concerns the OCL problem while the second one concerns the OSL problem.

6. APPROACH (APPR): This point out whether the filtering procedure (when applied) is developed either in finite ($F_{\mathbb{R}^N}$) or infinite dimensional spaces; the latter case presenting two possibilities, (F_{L_2}) or (F_{ℓ_2}), according to filtering in $L_2(\Omega)$ or in ℓ_2, respectively. The symbol (C) will stand for control (or controllability) problems, which are approached in $\mathbb{R}^N$.

7. MAIN GOAL: The final problem for which the optimal location problem is developed was characterized by the already posed notation SLE, OCL, and SCL.

Methods classification

The block diagram in figure 2 presents a

REFERENCE	INPUT	OBS. NOISE	BC	IC	#	APPR.	MAIN GOAL
[79] YU, SEINFELD (1973)	w	v	H	D	2	$F_{\mathbb{R}^N}$	
[23] CARAVANI, DI PILLO, GRIPPO (1975)	0	v	H	E	1	$E_{\mathbb{R}^N}$	SLE (group I)
[59] OMATU, KOIDE, SOEDA (1978)	w	v	H	S	2	$F_{\mathbb{R}^N}$	
[69] SAWARAGI, SOEDA, OMATU (1978)	w	v	H	S	2	$F_{\mathbb{R}^N}$	
[15] BENSOUSSAN (1972)	w	v	D,H	S	0	F_{L_2}	
[2] AIDAROUS, GEVERS, INSTALLÉ (1975)	w	v	H	S	2	F_{L_2}	
[10] AMOUROUX, BABARY, MALANDRAKIS (1978)	w	v	H	S	2	F_{ℓ_2}	SLE (group II)
[30] CURTAIN, ICHIKAWA (1978)	w	v	S,H	D	2	F_{L_2}	
[47] KUMAR, SEINFELD (1978)	w	v	H	S	2	F_{L_2}	
[57] NAKAMORI, MIYAMOTO, IKEDA, SAWARAGI (1980)	w	v	D,H	0	1	F_{L_2}	
[6] AMOUROUX, BABARY (1973)	u_d	0	H	D	1	C	
[7] AMOUROUX, BABARY (1975)	u_d	0	H	D	1	C	
[3] AIDAROUS, GEVERS, INSTALLÉ (1976)	u + w	v	H	S	1	C	OCL
[8] AMOUROUX, BABARY (1978)	u_d	0	H	D	1	C	
[9] AMOUROUX, BABARY (1979)	u_d	0	H	D	1	C	
[11] AMOUROUX, DI PILLO, GRIPPO (1976)	u	v	H	D	1/4	$F_{\mathbb{R}^N}$/C	
[33] EL JAI, (1977)	u	v	H	E	3/4	$E_{\mathbb{R}^N}$/C	
[27] COURDESSES (1978)	u_d	0	H	D	3/1	C	SCL
[42] ICHIKAWA, RYAN (1979)	u + w	v	H	D	1/1	F_{L_2}/C	
[54] MALANDRAKIS (1979)	u + w	v	D	S	2/1	F_{L_2}/C	

Table 1: Summary of model and method characteristics.

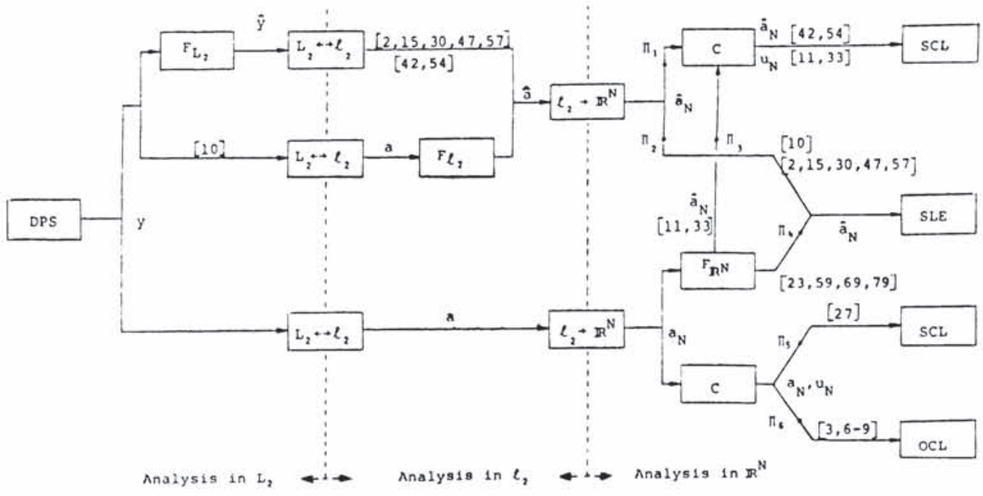

Fig. 2: A classification of OCL and OSL methods.

classification of methods in the light of N-modal approximation schemes. The main classifying factor concerns the different stages of the optimization procedure in which such approximations (or truncations) are required. In addition to the notation already posed in this paper, the following has also been adopted in the diagram of fig. 2.

$L_2 \leftrightarrow \ell_2$: Standing for the equivalent (infinite-dimensional) system representation, either in $L_2(\Omega)$ or ℓ_2, according to the eigenvector series expansion.

$\ell_2 \to \mathbb{R}^N$: Standing for the N-modal approximation; that is, the truncation of eigenvector series expansion in its first N terms.

Numbers between square brackets concern the references mentioned in table 1, and they point out the path which classifies the underlying method as follows:

Path Π_1: SCL methods using infinite-dimensional filtering.
Path Π_2: SLE methods using infinite-dimensional filtering.
Path Π_3: SCL methods using finite-dimensional filtering.
Path Π_4: SLE methods using finite-dimensional filtering.
Path Π_5: SCL methods approached in a deterministic environment.
Path Π_6: OCL methods approached in a deterministic environment.

5. COMMENTS AND CONCLUDING REMARKS

Several remarks and some conclusions can be drawn from what has been discussed in the preceding sections. A brief selection of basic topics which deserve to be emphasized will be presented in this final section.

1. Although this seems to be the first attempt to survey the several OCL and OSL methods for DPS, practical motivations for considering the problem were not addressed here. However such motivations can be found in the surveys [46,61,64,67] and books [20,66,68,76] mentioned in section 1, in connection to identificatio filtering, and control problems in DPS.

2. Little literature has been written about OCL and OSL, compared with what has been published in either identification, filtering or control of DPS.

3. In particular, more research is needed regarding the OSL problem for system identification (i.e., the SLI problem).

4. Gaussian distribution has always been assumed, for both input disturbance and/ or observation noise, when the DPS is supposed to operate in a stochastic environment.

5. The great majority of the methods reviewed here apply to linear models. More effort towards OCL and OSL methods for non-linear DPS should be attempted.

6. As already remarked here, N-modal is the most used approximation technique in OCL and OSL. Opposit to other areas in the DPS field (e.g., in DPS identification) finite-differences is not a very popular scheme, even among the methods which approximate the PDE to an ODE (or difference equation) thus reducing the DPS (modelled in an infinite-dimensional state space) to an LSP (modelled in a finite dimensional state space).

7. On the other hand, as in the whole DPS field, the question of the when to use approximation techniques does not seem to have a final answer yet. According to section 4, approximations have been applied either before or after optimization schemes. When the filtering problem was involved, it has been performed either in $L_2(\Omega)$, ℓ_2 or $\mathbb{R}^N$; but the control problem was generally developed in $\mathbb{R}^N$. In any case the OCL and OSL strategies were normally developed after applying approximation techniques.

8. Up to a few papers (e.g., see [33,49]), the optimal placement of an "a priori" fixed number of sensors and/or controllers has been considered. The problem of optimizing the number of sensors and/or controllers should receive more attention.

9. More research is also needed towards OCL for boundary controls.

10. The simulated results presented in the literature have generally been developed for DPS with one-dimensional spatial domain. Illustrative examples and experimental results considering two or three dimensional spatial domains would be welcome.

11. Perhaps it is already time to have some comparison of effectiveness of the different OCL and OSL methods. The classification introduced in section 4 can be viewed as a first step for a qualitative comparison. It can also be used as a starting-point for further works towards a quantitative comparison, since some different approaches for solving the OCL and OSL problem have been grouped according to their main structural characteristic.

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