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SIMILARITY TO CONTRACTIONS AND WEAK STABILITY

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Abstract: A survey on the problem of characterizing Hilbert-space operators that are similar to contractions is followed by an analysis on the relationship between similarity to a contraction and weak stability. This leads to the problem of characterizing the weakly stable unitary operators.

1. INTRODUCTION

Throughout this paper $\mathcal{H}$ and $\mathcal{H}'$ will stand for unitarily equivalent nontrivial complex separable Hilbert spaces, and $\ell_2(\mathcal{H}) = \oplus_{k=0}^{\infty}\mathcal{H}$ will denote the Hilbert space obtained by the direct sum of countably infinite copies of $\mathcal{H}$. Let $\mathcal{B}[\mathcal{H},\mathcal{H}']$ stand for the Banach space of all operators (i.e. bounded linear transformations) of $\mathcal{H}$ into $\mathcal{H}'$, and let $\mathcal{G}[\mathcal{H},\mathcal{H}']$ denote the class of all invertible operators from $\mathcal{B}[\mathcal{H},\mathcal{H}']$ (i.e. those which are also bounded below and map $\mathcal{H}$ onto $\mathcal{H}'$). Set $\mathcal{B}[\mathcal{H}] = \mathcal{B}[\mathcal{H},\mathcal{H}]$ and $\mathcal{G}[\mathcal{H}] = \mathcal{G}[\mathcal{H},\mathcal{H}]$ for short. Let $\mathcal{B}^+[\mathcal{H}]$ be the weakly closed convex cone of all (self-adjoint) nonnegative operators in $\mathcal{B}[\mathcal{H}]$, and set $\mathcal{G}^+[\mathcal{H}] = \mathcal{G}[\mathcal{H}] \cap \mathcal{B}^+[\mathcal{H}]$. Point spectrum, residual spectrum and continuous spectrum in $\mathcal{B}[\mathcal{H}]$ will be denoted by $\sigma_P(\cdot)$, $\sigma_R(\cdot)$ and $\sigma_C(\cdot)$; and $r(\cdot)$, $w(\cdot)$ and $||\cdot||$ will stand for spectral radius, numerical radius and (induced uniform) norm in $\mathcal{B}[\mathcal{H}]$; respectively. As usual, an upper star * will denote adjoint. T will always mean an operator in $\mathcal{B}[\mathcal{H}]$.

An operator T is said to be *weakly*, *strongly* or *uniformly stable* if the power sequence $\{T^n;\ n \geq 1\}$ converges weakly, strongly or uniformly to the null operator (i.e. if $T^n \xrightarrow{w} O$, $T^n \xrightarrow{s} O$, or $T^n \xrightarrow{u} O$), respectively. If the power sequence is bounded (recall that weak, strong and uniform boundedness are equivalent by the Banach-Steinhaus theorem), then T is called *power bounded* (i.e. if $\sup_n ||T^n|| < \infty$). According to the Gelfand-Beurling formula ($r(T) = \lim_n ||T^n||^{1/n}$) power boundedness, stability and the spectral radius are related as follows.

$$r(T) < 1 \iff T^n \xrightarrow{u} O \implies$$

$$T^n \xrightarrow{s} O \implies T^n \xrightarrow{w} O \implies \sup_n ||T^n|| < \infty \implies r(T) \leq 1.$$

An operator $T \in \mathcal{B}[\mathcal{H}]$ is *similar* to $C \in \mathcal{B}[\mathcal{H}']$ if $C = WTW^{-1}$ for some $W \in \mathcal{G}[\mathcal{H}, \mathcal{H}']$. Recall that similarity preserves stability, power boundedness, the spectrum and its parts $\sigma_P(\cdot)$, $\sigma_R(\cdot)$ and $\sigma_C(\cdot)$. By a *contraction* (a *strict contraction*) we mean an operator T such that $||T|| \leq 1$ ($||T|| < 1$).

PROBLEM 1. *Which operators are similar to contractions?*

Halmos has classified this as "one of the most difficult open problems of operator theory" [11 p.82]. The purpose of the present paper is to survey some specific aspects related to the above problem. Particularly, we shall be focusing on the relationship between similarity to a contraction and weak stability. Before going further let us recall that the following assertions are equivalent.

(a) T is similar to a contraction.
(b) There exists $Q \in \mathcal{G}^+[\mathcal{H}]$ such that $||QTQ^{-1}|| \leq 1$.
(c) There exists $Q \in \mathcal{G}^+[\mathcal{H}]$ such that $Q^2 - T^*Q^2T \in \mathcal{B}^+[\mathcal{H}]$.
(d) T^n is similar to a contraction for some integer $n \geq 1$.
(e) There exist a contraction $C \in \mathcal{B}[\mathcal{H}']$, $A \in \mathcal{B}[\mathcal{H}', \mathcal{H}]$ and $B \in \mathcal{B}[\mathcal{H}, \mathcal{H}']$ such that $T^n = AC^nB$ for every integer $n \geq 1$.
(f) There exist a contraction $C \in \mathcal{B}[\mathcal{H}']$, $A \in \mathcal{B}[\mathcal{H}', \mathcal{H}]$ and $B \in \mathcal{B}[\mathcal{H}, \mathcal{H}']$ such that $\sum_{n=0}^{\infty} ||AC^nB - T^n||^2 < \infty$.

The above equivalent conditions to similarity to a contraction have been verified in [9, 13, 14, 18].

2. POWER BOUNDEDNESS

Perhaps the history of Problem 1 has begun in 1947 when the following result, due to Nagy [24], was published.

PROPOSITION 1. *If $T \in \mathcal{G}[\mathcal{H}]$ and both T and T^{-1} are power bounded, then T is similar to a unitary operator.*

Note that the converse is trivially true, and that similarity to a unitary operator is a particular case of similarity to a contraction. In 1958, trying to remove the invertibility condition from the above proposition, Nagy [25] presented a solution to Problem 1 for a particular class of operators, which in turn solves Problem 1 for the finite-dimensional case.

PROPOSITION 2. *Every power bounded compact operator is similar to a contraction.*

Since similarity to a contraction implies power boundedness in general, the above proposition characterizes, within the class of all compact operators, those which are similar to a contraction: they are the power bounded operators. It was then that Nagy has posed the following question, which closes Halmos' 1963 "Glimpse into Hilbert Spaces" [7].

QUESTION 1. *Does the equivalence between power boundedness and similarity to a contraction still hold for noncompact operators?*

The answer was given by Foguel [6] in 1964 and reworked in a very ellegant way by Halmos [8] on the pages following Foguel's paper.

Answer 1. No. Let $S \in \mathcal{B}[\mathcal{H}]$ be a unilateral shift, which means that there exists an orthonormal basis $\{e_k;\ k \geq 0\}$ for $\mathcal{H}$ such that $Se_k = e_{k+1}$ for every $k \geq 0$. Let $P \in \mathcal{B}[\mathcal{H}]$ be the orthogonal projection onto the closed span$\{e_j\ :\ \ j = k^3;\ k \geq 1\}$. The following operator $F \in \mathcal{B}[\mathcal{H} \oplus \mathcal{H}]$ is power bounded but not similar to a contraction:

$$F = \begin{pmatrix} S^* & P \\ O & S \end{pmatrix}.$$

However, there are classes of not necessarily compact operators where the above equivalence between power boundedness and similarity to a contraction does hold. For instance, the class of all spectraloid operators. Recall that an operator T is *spectraloid* if $r(T) = w(T)$. T is *normaloid* if $r(T) = ||T||$, or equivalently if $w(T) = ||T||$. An operator is *hyponormal* if $TT^* \leq T^*T$. A *subnormal* operator is one that has a normal extension (i.e. one that is the restriction of a normal operator to an invariant subspace). An operator T is called *quasinormal* if it commutes with T^*T. These classes are related as follows.

$$\text{Normals} \subset \text{Quasinormals} \subset \text{Subnormals} \subset$$
$$\text{Hyponormals} \subset \text{Normaloids} \subset \text{Spectraloids.}$$

PROPOSITION 3. *Every power bounded spectraloid operator is similar to a contraction.*

Indeed, T is similar to a contraction whenever $w(T) \leq 1$ [27 p.95]. By going just one step down similarity to a contraction collapses to contraction itself. Actually, by recalling that T is normaloid if and only if $||T^n|| = ||T||^n$ for every integer $n \geq 1$, it follows that *every power bounded normaloid operator is a contraction.*

In light of the negative answer to Question 1, Halmos reformulated it in 1970 where power boundedness was replaced by polynomial boundedness. Thus the sixth problem in Halmos' "Ten Problems in Hilbert Space" [9] reads as follows: "is every polynomially bounded operator similar to a contraction?" Recall that an operator T is *polynomially bounded* if $\sup_p ||p(T)||/||p||_\infty < \infty$, where the supremum is taken over all polynomials p

and $||p||_\infty = \sup_{|\lambda|\le 1} |p(\lambda)|$. Since similarity to a contraction implies polynomial boundedness, what is actually being asked is whether polynomial boundedness is the solution of Problem 1. Some partial evidences towards an affirmative answer were presented by Chatage [2] in 1975. However, evaluating the progress on those ten problems posed in [9], Halmos reported in 1979 that there was no progress up to then regarding a possible equivalence between similarity to a contraction and polynomial boundedness [10]. Apparently another decade of 'ignorance' has already been completed. By the way, the operator F appearing in Answer 1 is not polynomially bounded, as verified by Lebow [23] in 1968.

3. **WEAK STABILITY**

Let us now have a somewhat different look at similarity to a contraction. First we note that weak stability and similarity to a contraction share some common properties. For instance, let Δ and Γ denote the open unit disc and unit circle in the complex plane centered at the origin, respectively, and consider the following propositions.

PROPOSITION 4. *If T is weakly stable, then T is power bounded and $\sigma_P(T) \cup \sigma_R(T) \subseteq \Delta$.*

PROPOSITION 5. *If T is similar to a contraction, then T is power bounded and $\sigma_R(T) \subseteq \Delta$.*

Since weak stability trivially implies $\sigma_P(T) \subseteq \Delta$, Proposition 4 is readily verified by recalling that the adjoint operation preserves weak stability, and that $\sigma_R(T) = \sigma_P(T^*)^* \backslash \sigma_P(T)$. Since similarity preserves power boundedness, as well as each part of the spectrum, Proposition 5 is also readily verified because $\sigma_R(T) \subseteq \{\lambda \in \mathbb{C}\colon\ |\lambda| < ||T||\}$ for any operator T (see e.g. [20]).

Recall that $r(T) \le 1$ whenever T is power bounded, and $r(T) < 1$ if and only if T is uniformly stable. Thus, according to Proposition 4, we may conclude: if the continuous spectrum does not intersect the unit circle (trivial example: compact operators), then the weak and uniform (and so strong) stability concepts coincide. In other words, T *is uniformily stable if and only if T is weakly stable and $\sigma_C(T) \cap \Gamma = \emptyset$.*

Next recall that $r(T) = \inf_{W\in\mathcal{G}[\mathcal{H},\mathcal{H}']} ||WTW^{-1}||$ [30], and hence an operator T is similar to a strict contraction if and only if $r(T) < 1$. Therefore uniform stability is equivalent to similarity to a strict contraction. A natural question, raised in [20], is whether the implications characterized by the above equivalence can survive (at least in one sense) the following relaxation: on the one hand change uniform stability to weak stability, and on the other hand replace similarity to a strict contraction by similarity to a contraction.

QUESTION 2. *Is every weakly stable operator similar to a contraction?*

If there exists a weakly stable operator not similar to a contraction, then it is necessarily a power bounded operator not similar to a contraction. The operator we have seen with this property was the operator F in Answer 1. However such an operator F is not weakly stable. Indeed, Foguel has shown in [6] that $\mathcal{Z}(T) \cap \mathcal{Z}(T^*)^{\perp} = \{0\}$ whenever T is similar to a contraction, where $\mathcal{Z}(T) := \{x \in H : T^n x \xrightarrow{w} 0\}$ and the upper symbol $^{\perp}$ denotes orthogonal complement (note: an operator T is weakly stable if and only if $\mathcal{Z}(T) = \mathcal{H}$). The operator F was deliberately built in [6] to fit the condition $\mathcal{Z}(F) \cap \mathcal{Z}(F^*)^{\perp} \neq \{0\}$, thus ensuring that it is not similar to a contraction. But this ensures that it is not weakly stable as well (for $\mathcal{Z}(F^*)^{\perp} \neq \{0\}$, so that $\mathcal{Z}(F^*) \neq \mathcal{H}$, and hence F^* is not weakly stable, or equivalently F is not weakly stable). Note that, since the power bounded operator F is neither weakly stable nor similar to a contraction, it also shows that the converses to Propositions 4 and 5 fail (because $\sigma_P(F) = \sigma_P(S^*) = \Delta$, $\sigma_R(F) = \sigma_R(S^*) = \emptyset$ and $\sigma_C(F) = \sigma_C(S^*) = \Gamma$).

A partial evidence towards a negative answer to Question 2 is supplied by the following proposition.

Proposition 6. *If a strongly stable operator is similar to a contraction, then it is similar to a part of a backward shift.*

Note that the converse is trivially true, so that a positive answer to Question 2 would lead to a universal model for strong stability (viz. operators that are similar to parts of backward shifts). Recall that a part of an operator is a restriction of it to an invariant subspace. The above proposition is a simple extension of the following result (see e.g. [4 p.23]): a strongly stable contraction is unitarily equivalent to a part of a backward shift. This in turn is a refinement of Rota's theorem [30] (if $r(T) < 1$, then T is similar to a part of a backward shift) due to de Branges and Rovnyak [1]. Note that the above mentioned backward shifts (i.e. adjoint of unilateral shifts) are generally of infinite multiplicity (they really are defined on $\ell_2(\mathcal{H})$).

A final answer to Question 2 has recently appeared in [19].

Answer 2. No. Not even strong stability implies similarity to a contraction. Take the operator $F \in \mathcal{B}[\mathcal{H} \oplus \mathcal{H}]$ defined in Answer 1. The following operator $S_F^* \in B[\ell_2(\mathcal{H} \oplus \mathcal{H})]$ is strongly stable but not similar to a contraction: $S_F^* x = \oplus_{k=0}^{\infty} F x_{k+1}$ for all $x = \oplus_{k=0}^{\infty} x_k \in \ell_2(\mathcal{H} \oplus \mathcal{H})$. S_F^* is the product of a backward shift of infinite multiplicity by the infinite direct sum $\oplus_{k=0}^{\infty} F$. Thus it can be thought of as a backward shift of infinite multiplicity that has been constantly weighted by F, which is identified with the following infinite matrix of operators

$$S_F^* = \begin{pmatrix} O & F & & & \\ & O & F & & \\ & & O & F & \\ & & & \ddots & \\ & & & & \end{pmatrix}.$$

It is obvious that similarity to a contraction does not imply weak stability; just take a contraction with an eigenvalue in the unit circle (e.g. the identity). Therefore a suitable converse to Question 2 should read as follows [20].

Question 3. *Does similarity to a contraction imply weak stability for operators with point spectrum in the open unit disc?*

If T is a contraction, then $\mathcal{Z} = \{x \in \mathcal{H} : \; T^n x \xrightarrow{w} 0\}$ is a subspace (i.e. a closed linear manifold) which reduces T, and T is unitary on $\mathcal{Z}^\perp$. This is Foguel's decomposition [5]: a contraction is the direct sum of a weakly stable contraction and a unitary operator. However, the largest reducing subspace of a contraction T on which it is unitary is not $\mathcal{Z}^\perp$ in general, but $\mathcal{K} = \{x \in \mathcal{H} : \; ||T^n x|| = ||T^{*n} x|| = ||x||, \;\; \forall n \geq 1\}$, so that T is completely nonunitary on $\mathcal{K}^\perp$ (an operator is *completely nonunitary* if the restriction of it to any nonzero reducing subspace is not unitary). This is Nagy-Foias-Langer decomposition [22, 26]: a contraction is uniquely the direct sum of a completely nonunitary contraction and a unitary operator. According to the above decompositions *a completely nonunitary contraction is weakly stable* (for $\mathcal{Z}^\perp \subseteq \mathcal{K}$, so that $\mathcal{K}^\perp \subseteq \mathcal{Z}$). By using Foguel's decomposition it has been verified in [20] that the following question is equivalent to Question 3.

Question 3'. *Is every unitary operator with empty point spectrum weakly stable?*

Perhaps the first unitary operator without eigenvalues that comes to one's mind is a bilateral shift (see e.g. [29 ch.3]). However, a bilateral shift is weakly stable. Actually, *for any weighted shift* (bilateral or unilateral of arbitrary multiplicity) *weak stability is equivalent to power boundedness, which in turn is equivalent to similarity to a contraction* [31] (caution: the operator S_F^* in Answer 2 is not a weighted backward shift of infinite multiplicity).

Recall that a unitary operator is *absolutely continuous* (*singular, singular-continuous, singular-discrete*) if its spectral measure is absolutely continuous (singular, singular-continuous, singular-discrete) with respect to normalized Lebesgue measure on the unit circle. Every unitary operator is uniquely the direct sum of an absolutely continuous unitary and a singular unitary (see e.g. [4 p.55, 56]). But an absolutely continuous unitary operator is similar to a completely nonunitary contraction [15], so that *an absolutely continuous unitary operator is weakly stable* (reason: similarity preserves stability). Hence Question 3' actually asks whether a singular unitary operator with empty point spectrum is weakly stable. Now recall that a singular unitary operator is the direct sum of a singular-continuous unitary and a singular-discrete unitary, and that the absence of point spectrum implies that its singular-discrete direct summand is missing. Therefore Question 3 can be refined a bit further than Question 3', being equivalent to the following one as well.

Question 3". *Is every singular-continuous unitary operator weakly stable?*

When formulating Question 3 we cared of ruling out contractions with eigenvalues in the unit circle, so that unitary diagonals have been put aside from Question 3', since they are certainly not weakly stable. Question 3" also rules out bilateral shifts or any direct summand of them, since these are precisely the absolutely continuous unitary operators (see e.g. [4 p.55, 56]), and hence weakly stable.

Before answering Question 3 let us recall that the spectral measure of a unitary operator is called *pure* if it is either absolutely continuous, singular-continuous or singular-discrete with respect to normalized Lebesgue measure on the unit circle. Putting it another way, if two direct summands are necessarily missing in the decomposition of a unitary operator as a direct sum of an absolutely continuous unitary, a singular-continuous unitary, and a singular-discrete unitary.

Answer 3. No. There are singular-continuous unitary operators that are not weakly stable. For instance, let μ be the normalized Lebesgue measure on Γ, set $\mathcal{H} = \mathcal{L}_2(\mu)$, and consider the following unitary operator $U \in \mathcal{B}[\mathcal{L}_2(\mu)]$:

$$(Uf)(z) = z^q f(\gamma z) \qquad \text{a.e. on} \quad \Gamma$$

for all $f \in \mathcal{L}_2(\mu)$, where q is a sufficiently small nonzero rational (e.g. $0 < |q| \leq 1/12$) and γ is an irrational in Γ (i.e. $\gamma = e^{2\pi i \alpha}$ with $\alpha \in [0, 1)$ irrational). It has recently been verified in [3] that U is not a singular-discrete unitary and that $\{U^n;\ n \geq 1\}$ has a subsequence, say $\{U^{n_k};\ k \geq 1\}$, such that $0 < \inf_k |\langle U^{n_k} 1; 1\rangle|$. Therefore U is not weakly stable, and hence it is not an absolutely continuous unitary as well. However, the spectral measure of U is pure (see e.g. [12]), so that U must be a singular-continuous unitary.

To close this section we note that the above technique used to give a negative answer to Question 3 has a peculiar aspect: the singular-continuous unitary operators it supplies are born weakly unstable. Thus it may not provide an insight into the answer to the next natural question, but it certainly suggests the question: is every singular-continuous unitary operator weakly unstable? In other words, are the weakly stable unitary operators precisely the absolutely continuous ones? Equivalently, is a unitary operator weakly stable if and only if it is a bilateral shift or a direct summand of it?

4. **FINAL REMARKS**

The uniform stability problem (i.e. the characterization of all uniformly stable operators) is certainly solved (recall: T is uniformly stable if and only if $r(T) < 1$, which in turn is equivalent to similarity to a strict contraction). Moreover, there exists in current literature a huge collection of equivalent conditions for uniform stability (see e.g. [16, 17, 18, 21] and the references therein). However, the same problem in weaker topologies still remains unsolved. As far as weak stability is concerned (which operators are weakly stable?) we have made an attempt here to survey some aspects of the problem by trying to interweave it with Problem 1.

Regarding strong stability the problem presents an extra complexity: opposite to uniform and weak stabilities, strong stability is not preserved by the adjoint operation. Furthermore (and this is perhaps what is actually behind the above remark), the role played by strong stability in the invariant subspace problem (witness: the $C_{\cdot\cdot}$ classes of Nagy and Foias [27]) ranks it in a much more delicate status than weak stability; even though we can already classify strong stability among specific classes of operators. For instance, Putnam has shown in [28] that a cohyponormal operator (i.e. one whose adjoint is hyponormal) is strongly stable if it is a completely nonunitary contraction (note: the converse holds even for a normaloid operator).

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